

Relative Limiting Error

SUM

$$X = X_1 + X_2$$

$$\frac{dX}{X} = \frac{dX_1}{X} + \frac{dX_2}{X}$$

$$\frac{dX}{X} = \frac{dX_1}{X} \cdot \frac{x_1}{x_1} + \frac{dX_2}{X} \cdot \frac{x_2}{x_2}$$

$$\frac{dX}{X} = \frac{dX_1}{x_1} \cdot \frac{x_1}{X} + \frac{dX_2}{x_2} \cdot \frac{x_2}{X}$$

$\pm \delta x_1$ or $\pm \delta x_2$ Represent limiting error

$$\frac{\delta X}{X} = \pm \left(\frac{\delta X_1}{x_1} \cdot \frac{x_1}{X} + \frac{\delta X_2}{x_2} \cdot \frac{x_2}{X} \right)$$

Difference

$$X = X_1 - X_2$$

$$\frac{dX}{X} = \frac{dX_1}{X} - \frac{dX_2}{X}$$

$$\frac{dX}{X} = \frac{dX_1}{X} \cdot \frac{x_1}{x_1} - \frac{dX_2}{X} \cdot \frac{x_2}{x_2}$$

$$\frac{dX}{X} = \frac{dX_1}{x_1} \cdot \frac{x_1}{X} - \frac{dX_2}{x_2} \cdot \frac{x_2}{X}$$

$+\delta x_1$ and $-\delta x_2$ Worst discrepancy case

$$\frac{\delta X}{X} = \pm \left(\frac{\delta X_1}{x_1} \cdot \frac{x_1}{X} + \frac{\delta X_2}{x_2} \cdot \frac{x_2}{X} \right)$$

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Product

$$X = X_1 \cdot X_2$$

$$\log X = \text{Log}X_1 + \text{Log}X_2$$

$$\frac{1}{X} = \frac{1}{X_1} \frac{dX_1}{dX} + \frac{1}{X_2} \frac{dX_2}{dX}$$

$$\frac{dX}{X} = \frac{dX_1}{X_1} + \frac{dX_2}{X_2}$$

$\pm \delta x_1$ or $\pm \delta x_2$ Represent limiting error

$$\frac{\delta X}{X} = \pm \left(\frac{\delta X_1}{x_1} + \frac{\delta X_2}{x_2} \right)$$

Devision

$$X = X_1 / X_2$$

$$\log X = \text{Log}X_1 - \text{Log}X_2$$

$$\frac{1}{X} = \frac{1}{X_1} \frac{dX_1}{dX} - \frac{1}{X_2} \frac{dX_2}{dX}$$

$$\frac{dX}{X} = \frac{dX_1}{X_1} - \frac{dX_2}{X_2}$$

$+\delta x_1$ and $-\delta x_2$ Worst discrepancy case

$$\frac{\delta X}{X} = \pm \left(\frac{\delta X_1}{x_1} + \frac{\delta X_2}{x_2} \right)$$

Relative Limiting Error

Power

$$X = X_1^n$$

$$\log X = n \text{Log} X_1$$

$$\frac{1}{X} = n \cdot \frac{1}{X_1} \frac{dX_1}{dX}$$

$$\frac{dX}{X} = n \frac{dX_1}{X_1}$$

$$\pm \delta x_1$$

$$\frac{\delta X}{X} = \pm n \cdot \frac{\delta X_1}{x_1}$$

Composites

$$X = X_1^n \cdot X_2^m$$

$$\log X = n \text{Log} X_1 + m \text{Log} X_2$$

$$\frac{1}{X} = n \frac{1}{X_1} \frac{dX_1}{dX} + m \frac{1}{X_2} \frac{dX_2}{dX}$$

$$\frac{dX}{X} = n \frac{dX_1}{X_1} + m \frac{dX_2}{X_2}$$

$\pm \delta x_1$ and $\pm \delta x_2$ Represents limiting error

$$\frac{\delta X}{X} = \pm \left(n \frac{\delta X_1}{x_1} + m \frac{\delta X_2}{x_2} \right)$$

Numerical

Question:

Resistance of a circuit found by measuring current and power fed into it. Find limiting error in measurement when power and current deviates by $\pm 1.5\%$ and current by $\pm 1.0\%$

$$\text{Resistance} = \frac{\text{Power}}{(\text{Current})^2}$$

$$R = P I^{-2}$$

$$\frac{\delta R}{R} = \pm \left(\frac{\delta P}{P} + 2 \frac{\delta I}{I} \right)$$

$$\frac{\delta R}{R} = \pm (1.5 + 2 * 1.0) = \pm 3.5\%$$